## **Harmonic Analysis**

## Abstract

When transmitting radio or television, the information in low-frequency signals (e.g. sound waves) will have to be converted into a high-frequency signals. This can be achieved in several ways, for example amplitude modulation (AM), frequency modulation (FM) or phase modulation. In this experiment we study the behavior of amplitude modulation, which mathematically is the multiplication of the input signal with a carrier signal with a known high frequency. For a sinusoidal carrier signal (U<sub>c</sub> = A + cos  $\omega_c t$ ) and a sinusoidal input signal (U<sub>M</sub> = B + cos  $\omega_M$ t) this gives an output signal of U =  $\alpha$  [A cos  $\omega_M t$  + B cos  $\omega_C t$  +  $\frac{1}{2}$ (cos ( $\omega_{_{C}}-\omega_{_{M}})$  t + cos ( $\omega_{_{C}}$  +  $\omega_{_{M}})$  t) ], where  $\alpha$  is the amplification. The resulting signal has four peaks in frequency space, as illustrated in the figure below, which is the Fourier transform of the output signal measured with an oscilloscope.



Frequency spectrum of modulated signal

In the case of balanced modulation we add a signal with a constant voltage to the input and carrier signal, until A and B are equal to zero. In this case the output signal will no longer have the frequencies of the input and carrier signal, but only two peaks in frequency space at  $(\omega_c - \omega_M)$  and  $(\omega_c + \omega_M)$ . The frequencies in an original low frequency signal can be reproduced from such a balanced modulation by measuring the modulated signal in the frequency domain and calculating the offsets from the carrier frequency. In this experiment we study the effect of varying A and B on the output signal for  $\omega_c = 2.2$  MHz and  $\omega_M = 0.1$  MHz



A second goal in this experiment is to use a study of the output of a modulated signal, to retrieve the frequency domain of an input signal. The setup for such a measurement is shown in the diagram below:



We use balanced modulation (i.e A=B=0) to modulate a carrier signal with a measured frequency  $f_c$  with an input signal with an unknown frequency distribution  $f_M$ . By running the output signal of the modulater through a frequency filter, which only keeps the signal at 100 kHz and a rectifier, we can measure the amplitude of the modulated signal at 100 kHz. By varying the frequency of the carrier signal ( $f_c$ ) the output voltage of the rectifier will only be non-zero when  $f_c + f_M = 100$  kHz or  $f_c - f_M = 100$ kHz. So the measured dependence of the output voltage on the frequency of the carrier signal can be used to reproduce the original signal in frequency space.